EMS – IAMP Summer School in Mathematical Physics

Universality, Scaling Limits and Effective Theories

July 11 – 15, 2016, Roma (Italy)
“Sapienza” Università di Roma

Programme & Abstracts

Monday 11/07/2016

08:30 – 09:00 Opening

09:00 – 10:30 Y. AVRON (lect. 1)

10:30 – 11:00 Coffee Break

11:00 – 12:30 H. DUMINIL-COPIN (lect. 1)

12:30 – 14:00 Lunch Break

14:00 – 15:30 A. GIULIANI (lect. 1)

15:30 – 16:00 Break

16:00 – 17:00 R. BAUERSCHMIDT (lect. 1)

Tuesday 12/07/2016

08:45 – 10:15 Y. AVRON (lect. 2)

10:15 – 10:45 Coffee Break

10:45 – 11:45 P. CAPUTO

11:45 – 12:00 Break

12:00 – 13:00 H. DUMINIL-COPIN (lect. 2)

13:00 – 14:30 Lunch Break

14:30 – 16:00 P. PICKL (lect. 1)

16:00 – 16:30 Coffee Break

16:30 – 18:00 R. BAUERSCHMIDT (lect. 2)

Wednesday 13/07/2016

08:45 – 10:15 A. GIULIANI (lect. 2)

10:15 – 10:45 Coffee Break

10:45 – 11:45 N. ROUGERIE

11:45 – 12:00 Break

12:00 – 13:00 Y. AVRON (lect. 3)

13:00 – 14:30 Lunch Break

14:30 – 16:00 P. PICKL (lect. 2)

16:00 – 16:30 Coffee Break

16:30 – 18:00 H. DUMINIL-COPIN (lect. 3)

Thursday 14/07/2016

08:45 – 10:15 A. GIULIANI (lect. 3)

10:15 – 10:45 Coffee Break

10:45 – 11:45 E. PRODAN

11:45 – 12:00 Break

12:00 – 13:00 P. PICKL (lect. 3)

13:00 – 14:30 Lunch Break

14:30 – 16:00 R. BAUERSCHMIDT (lect. 3)

16:00 – 16:30 Coffee Break

16:30 – 18:00 H. DUMINIL-COPIN (lect. 4)

Friday 15/07/2016

09:00 – 10:30 Y. AVRON (lect. 4)

10:30 – 11:00 Coffee Break

11:00 – 12:30 P. PICKL (lect. 4)

12:30 – 14:00 Lunch Break

14:00 – 15:30 R. BAUERSCHMIDT (lect. 4)

15:30 – 16:00 Break

16:00 – 17:00 A. GIULIANI (lect. 4)

Titles and Abstracts

Courses

Adiabatic Quantum Transport

Y. AVRON
(Technion, Haifa, Israel)

The Renormalisation Group Approach to Spin Systems

R. BAUERSCHMIDT
(Harvard University, Cambridge, US)

This mini-course introduces a rigorous renormalisation group approach to classical spin systems, in the spirit of Wilson. The renormalisation group is a general conceptual framework that explains critical behaviour and its universality in terms of an associated dynamical system and its stability. Specifically, it will be shown how universal logarithmic corrections arise in the 4-dimensional $|\varphi|^4$ model.

Random Current Representation of the Ising Model

H. DUMINIL-COPIN
(Université de Genève, Geneva, Switzerland)

This mini-course will be devoted to the study of the Ising model through its random current representation. This representation provides precise information on truncated spin correlations. We plan to show how this information can be used to study the critical behavior of the model on Z^d.

The Quantum Hall Effect in Interacting Lattice Electron Systems

A. GIULIANI
(Università degli Studi Roma Tre, Rome, Italy)

In this course I will discuss the quantization properties of the Hall conductance for a class of interacting electron systems on two-dimensional lattices, which includes the Hubbard-Haldane model and the Hubbard-Hofstadter model. I will first review the notion of Kubo conductivity and illustrate it, by explicitly computing the transverse conductivity in a prototypical example, the non-interacting Haldane model, thus constructing its 'topological phase diagram'. Next, I will explain how to compute the conductivity in the weakly interacting case and how to prove its universality, i.e., its exact independence from the interaction strength. I will discuss the key ingredients involved in the proof, namely: (1) determinant bounds for the fermionic perturbation theory, (2) a reconstruction theorem for the Kubo conductivity (from the imaginary to the real time), (3) the use of lattice Ward Identities and of the Schwinger-Dyson equation.

Derivation of Effective Equations for Many Particle Systems

P. PICKL
(LMU München, Munich, Germany)

For the description of many particle systems one often uses effective equations, in particular in numerical simulations. A prominent example is the Hartree Fock equations which is used to describe systems of many Fermions. In some situations these effective equations can be derived from a microscopic system, for example the Schrödinger equation for many interacting Fermions. "Derived" means, that the effective description holds in good approximation when the particle number is large. In recent years there has been a lot of progress in deriving effective equations for many particle systems. Those developments shall be addressed in the class. We will derive effective equations both for quantum mechanical and classical systems.

Invited Talks

A Dynamical Phase Transition in Random Lattice Triangulations

P. CAPUTO
(Università degli Studi Roma Tre, Rome, Italy)

We consider lattice triangulations, i.e., triangulations of the integer points in a polygon in Euclidean plane. Our focus is on random triangulations in which a triangulation σ has weight $\lambda^{|\sigma|}$, where λ is a positive real parameter and $|\sigma|$ is the total length of the edges in σ. Empirically, this model exhibits a phase transition at $\lambda = 1$ (corresponding to the uniform distribution): for $\lambda < 1$ distant edges behave essentially independently, while for $\lambda > 1$ very large regions of aligned edges appear. We substantiate this picture as follows. For $\lambda < 1$ sufficiently small, we show that correlations between edges decay exponentially with distance (suitably defined), and also that the Glauber dynamics (the local Markov chain based on flipping edges) is rapidly mixing (in time polynomial in the number of edges in the triangulation). By contrast, for $\lambda > 1$ we show that the mixing time is exponential. For thin rectangular regions we obtain sharp mixing time bounds for all values of $\lambda < 1$.
This is joint work with F. Martinelli, A. Sinclair and A. Stauffer.

The Anderson Localization-Delocalization Transition in IQHE and Topological Insulators

E. PRODAN
(Yeshiva University, New York City, US)

One of the most interesting effects in condensed matter physics is the Anderson localization due to disorder. An Anderson insulator can be broadly defined as a condensed matter system with the Fermi level located in the essential spectrum and with vanishing diagonal transport coefficients. For a long time, it was believed that, excepting a handful of cases, all lower dimensional quantum systems are in fact Anderson insulators in normal laboratory conditions, where disorder is un-avoidable. In other words, the entire spectrum is Anderson localized, hence the Anderson localization-delocalization transition is completely absent when the Fermi level is combed over the spectrum, by gating techniques for example. One of the earlier known exceptions to this rule is the Integer Quantum Hall Effect (IQHE) in dimension 2, where a sequence of Anderson localization-delocalization transitions can be observed experimentally (the famous plateau-plateau transitions). The discovery of Topological Insulators proved that this exception is not singular and in fact there are vast classes of condensed matter systems which display the Anderson transition. In this talk, I will first review the experimental characterizations of these transitions in such condensed matter systems. I will then present some exactly-solvable models where the phase diagram can be computed explicitly and I will try to convince you that the existence of a sharp phase boundary has topological origins. The rest of the talk will focus on numerical algorithms based on operator algebras which enabled us unprecedented insight into the Anderson localization-delocalization transition. In particular, it enabled us to map the current-current correlation functions near a topological Anderson transition and to study its critical scaling behavior.

Emergent Anyons in Quantum Hall Physics

N. ROUGERIE
(Université Grenoble 1 & CNRS, Grenoble, France)

Anyons are by definition particles with quantum statistics different from those of bosons and fermions. They can occur only in low dimensions, 2D being the most relevant case for this talk. They have hitherto remained hypothetical, but there is good theoretical evidence that certain quasi-particles occurring in quantum Hall physics should behave as anyons.
I shall consider the case of tracer particles immersed in a so-called Laughlin liquid. I will argue that, under certain circumstances, these become anyons. This is made manifest by the emergence of a particular effective Hamiltonian for their motion. The latter is notoriously hard to solve even in simple cases, and well-controlled simplifications are highly desirable. I will discuss a possible mean-field approximation, leading to a one-particle energy functional with self-consistent magnetic field.
Joint work with Douglas Lundholm.